



Naum Il'ich Feldman

Born: 26 Nov 1918 in Melitopol, Zaporozhye oblast, Ukraine

Died: 20 April 1994 in Moscow, Russia

Naum Il'ich Feldman graduated from middle school in 1936 and entered the Faculty of Mathematics and Mechanics at the University of Leningrad. His studies, specialising in number theory and supervised by R O Kuz'min, were completed by 1941 when he graduated from Leningrad. However by this time World War II had been taking place for around 18 months and soon events would see Feldman involved in the war.

At the start of World War II Russia had been an ally of Germany but Hitler, after highly successful military campaigns in the west, turned his attention to attacking Russia. He conceived Operation Barbarossa and began to build up army divisions to carry out the attack. Barbarossa was launched on 22 June 1941 and the German armies rapidly moved east with the aim of reaching Moscow in eight weeks. Feldman was called up to serve in the Russian army in June and by October 1941 he was totally involved in the battle against the advancing German troops. By December 1941 the Germans ringed Moscow but now, partly due to the severe winter weather, their advance was halted. The Russians began to counter-attack the German armies around Moscow and Feldman saw action for the first time in this bitter fighting. He was later awarded a medal "For the Defence of Moscow".

Given the high Russian casualties during World War II, it is quite remarkable that Feldman survived the war. He saw action in many parts and near the end of the war he was involved in the two-month siege of Königsberg which ended in April 1945 with the city virtually destroyed. For his part in this siege Feldman was awarded a medal "For the Taking of Königsberg". He received many other decorations for his war service, including the order of the "Red Star", and "The Fatherland War". He also received medals "For War Service" and "For Victory over Germany".

At the end the war Feldman was with the Russian armies in East Prussia. Once he was demobbed from the army he returned to Moscow where, in 1946, he registered as a research student in the Institute of Mathematics of Moscow University. His Ph.D. studies were supervised by Aleksandr O Gelfond and his thesis was submitted in 1949. Since so much of Feldman's later work was based on the work he undertook for this thesis we give a brief overview of its main results at this point. In 1873 Hermite published the first proof that e is a transcendental number. This result was strengthened by Borel in 1899 when he proved a lower bound for $P(e)$, where P is a polynomial with integer coefficients, depending on the maximum modulus of the integer coefficients of P . Gelfond, Feldman's supervisor, had extended Borel's result to numbers of the form α^β , where α, β are algebraic numbers. Feldman proved in his thesis Borel type results (called the measure of transcendence) for logarithms of algebraic numbers, obtaining estimates for the lower bound depending (as did Gelfond) on both the degree of P and the maximum modulus of its coefficients.

After the award of his Ph.D., Feldman was sent to the Ufimskii Oil Institute where he was appointed as Head of the Department of Mathematics. He worked there for four years from 1950 then he went to the Moscow Institute of Geological Prospecting, teaching there from 1954 to 1961. He then went to Moscow State University where he was appointed as a Reader in the Department of Mathematical Analysis. He was awarded a D.Sc in 1974 and was promoted to Professor in 1980.

In addition to his work on the measure of transcendence of numbers, Feldman also produced many results strengthening Liouville's theorem on the rational approximation of algebraic numbers. The power of Feldman's results in this area lies in the fact that he was able to give effective constants rather than just prove the existence of such constants. In *An effective power sharpening of a theorem of Liouville* in 1971 he proved the following theorem:-

Let α be an algebraic number of degree $n \geq 3$; then there exist effective positive constants a and C , depending only on α , such that $|\alpha - p/q| > Cq^{-a}$ for all rational integers p, q with $q > 0$.

These two themes are, of course, closely connected. For example in 1960 Feldman published two papers *The measure of transcendency of the number π* and *Approximation by algebraic numbers to logarithms of algebraic numbers* which were reviewed together by Mahler: - *In these two important papers, which are closely connected, measures of transcendency for π and $\log \alpha$ (α algebraic) are found which are far better than any obtained before. ... The proofs in both papers use similar ideas. They are based on Dirichlet's Schubfachprinzip and on two interpolation formulae...*

In 1982 Feldman published a 312 page text *Hilbert's seventh problem*. Hilbert's seventh problem asked for a proof of the transcendence of a to the power b when a is an algebraic number and b is an irrational algebraic number.

Loxton writes in a review of this book:-

This book is a leisurely account of the mathematics which has grown out of the attempts to solve [Hilbert's seventh problem]. The first chapter deals with the early history and the work of Hermite and Lindemann on the exponential function. Then follows a treatment of the methods of Gelfond and Schneider which led to the solution of Hilbert's seventh problem. Further applications of these methods to the arithmetic properties of elliptic functions, transcendence measures and algebraic independence are also given. The last part of the book describes Alan Baker's work on linear forms in the logarithms of algebraic numbers and its applications to Diophantine equations and to the determination of imaginary quadratic fields with class number 1 or 2. Most of this material might now be called "classical". However, the author includes frequent remarks on recent work in transcendence theory....

In [2] the authors write of Feldman as a teacher: - *Naum Il'ich was a gifted teacher and stood out for his great teaching skill. He loved to give lectures, to run seminars, to work with both undergraduate and postgraduate students. Many students did their coursework and diploma work under his supervision, and sic of them completed their Ph.D.s. He lectured uninterruptedly for 45 year, until his last day, to students of mathematics, devoting great power to his teaching.*

Naum Il'ich stood out by his great integrity, high principles, goodness and benevolence. He was highly respected by his undergraduate and postgraduate students, his colleagues in the department and faculty, and many mathematicians, who will always remember him as a well-known teacher, an outstanding lecturer, and a remarkable person.

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[<http://www-history.mcs.st-andrews.ac.uk/Biographies/Feldman.html>]